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Tripartite states' Bell-nonlocality sudden death in an environmental spin chain

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Abstract

The tripartite nonlocality is investigated by the extent of violation of the Bell inequality in a three-qubit system coupled to an environmental Ising spin chain. In the weak-coupling region, we show that the tripartite Bell-inequality violations can be fully destroyed in a finite time under decoherence induced by the coupling with the spin environment. In addition, how the environment affects the Bell-nonlocality sudden death is demonstrated.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

Entanglement is one of the most essential features in quantum mechanics and has received much attention in many fields of physics in both theoretical aspects and experimental ones [1]. It is a pure quantum correlation without classical counterpart and has been regarded as a basic resource in quantum technologies such as quantum teleportation and quantum cryptography [2]. As is known, a realistic system is surrounded by an environment, and the unavoidable coupling between them will lead to decoherence of the system, which is the main obstacle to realize quantum computing and quantum information processing. Much effort has been devoted to the study of the entanglement evolution under the influence of the environment [3–5]. It is pointed out by Yu and Eberly [3] that the entanglement of a pair of qubits exposed to local noisy environments can fall abruptly to zero within a finite time. Such a surprising phenomenon is termed entanglement sudden death (ESD). Due to its intrinsic and practical interest, ESD has attracted considerable attention and much progress has been made [6, 7]. However, the study of ESD has previously been limited in bipartite systems, for the reason that the measure of multipartite entanglement is a difficult problem which is far from being understood, and there is still no efficient general method to evaluate the quantum entanglement, especially for mixed states. For the multipartite system, the nonlocal behavior can be measured by the extent of the violation of the Bell-type inequality. Recently, the effect of Bell-nonlocality sudden death (BNSD) was indicated by the demonstration [8] that multipartite Bell-inequality violations could be fully destroyed in a finite time in three-qubit systems subject to basis-dependent multi-local asymptotic dephasing noise. A great deal of interest has been focused on the study of BNSD in tripartite systems, because the exhibition of BNSD illuminates the quantum–classical transition, quantum measurement and quantum information processing where joint-state coherence and nonlocality are considered significant. By following the methodology of [8], several researchers have discussed the destruction of nonlocality as measured by the extent of violation of the tripartite Bell-type inequality in finite time in various models [9–11].

More recently, there has been a growing interest in the study of decoherence induced by spin environments [12–17]. Cucchietti *et al* [13] examined the decoherence of a central spin interacting with a collection of independent spins. In the practical situation, particles in the environment may have interactions with each other. Consequently, the entanglement evolution in a correlated environment should be considered. Quan *et al* [14] have investigated the decoherence induced by a correlated Ising spin chain. They showed that the decoherence was best enhanced by the quantum phase transition of the surrounding system. Sun *et al* [15] considered two spins coupled to the Ising spin chain in a transverse field to reveal the effect of the correlated environment on the entanglement dynamics of the two spins. In addition, Ma *et al* [16] studied the entanglement evolution of three-qubit quantum states in a quantum-critical environment. But the study of tripartite nonlocality in a correlated environment is still an open issue.

Motivated by these, we turn to extend the analysis of BNSD in a three-qubit system, which is coupled to an Ising spin chain in a transverse field. Different from the previous studies of BNSD [8, 18] in which all interactions are local, we actually consider BNSD under the nonlocal nature of external noise in this paper. The nonlocality is demonstrated in a class of initially Bell-nonlocal pure states of tripartite systems, namely, the maximally entangled (GHZ) state. We also study the influence of the system–environment coupling, the strength of the transverse field and the size of the environmental Ising chain on the Bell nonlocality.

The paper is organized as follows. In section 2, we introduce the model of the three-qubit system coupled to a transverse Ising spin chain. By exactly diagonalizing the Hamiltonian, we give an expression of the reduced density matrix of the three-spin system. In section 3, the nonlocal behavior of the GHZ state is studied by analyzing the extent of violation of Bell-type inequality. The effect of BNSD is demonstrated. Finally, we give a short conclusion.

2. Model

We consider a three-qubit system coupled to an environmental spin chain, which is described by a one-dimensional transverse Ising model. The corresponding Hamiltonian is given by

$$H = H_E^{\omega} + H_I, \tag{1}$$

where (we take $\hbar = 1$)

$$H_E^{\omega} = \sum_j^L \sigma_j^x \sigma_{j+1}^x + \omega \sum_j^L \frac{\sigma_j^z}{2}, \tag{2a}$$

$$H_I = \left(\frac{g_A}{2}\sigma_A^z + \frac{g_B}{2}\sigma_B^z + \frac{g_C}{2}\sigma_C^z \right) \sum_j^L \frac{\sigma_j^z}{2}, \tag{2b}$$

with H_E^{ω} describing the Hamiltonian of the environmental Ising chain, and H_I denoting the interaction between the central three-qubit spins and the environmental Ising chain.

The parameter ω characterizes the strength of the transverse field, and g_l $(l = A, B, C)$ denote the coupling constants between the environment and the three spins. The operators σ_l $(l = A, B, C)$ and σ_j^α $(\alpha = x, y, z)$ are the Pauli matrices of the central three qubits and the jth site of the environmental spin chain, respectively, and the total number of spins in the Ising chain is L. The sum over j goes from 1 to L for periodic boundary conditions, where we assume that $\sigma_{L+1}^\alpha = \sigma_1^\alpha$.

In order to diagonalize the Hamiltonian, we rewrite H in the following form:

$$\begin{aligned} H &= \sum_{\mu\mu'} |\phi_\mu\rangle\langle\phi_\mu|H|\phi_{\mu'}\rangle\langle\phi_{\mu'}| \\ &= \sum_{\mu\mu'} |\phi_\mu\rangle\langle\phi_{\mu'}|\big(\langle\phi_\mu|H_I|\phi_{\mu'}\rangle + \langle\phi_\mu|H_E^\omega|\phi_{\mu'}\rangle\big), \end{aligned} \tag{3}$$

where $|\phi_\mu\rangle$ $(\mu = 1, \ldots, 8)$ is the μth eigenstate of the operator $\sum_l g_l\sigma_l^z\big/2$, and the corresponding eigenvalues can easily be obtained in the three-qubit subspace:

$$\begin{aligned} E_1 &= \frac{g_A + g_B + g_C}{2}, & E_2 &= \frac{g_A + g_B - g_C}{2}, \\ E_3 &= \frac{g_A - g_B + g_C}{2}, & E_4 &= \frac{g_A - g_B - g_C}{2}, \\ E_5 &= \frac{-g_A + g_B + g_C}{2}, & E_6 &= \frac{-g_A + g_B - g_C}{2}, \\ E_7 &= \frac{-g_A - g_B + g_C}{2}, & E_8 &= \frac{-g_A - g_B - g_C}{2}. \end{aligned} \tag{4}$$

We note that $\big[g_A\sigma_A^z + g_B\sigma_B^z + g_C\sigma_C^z, \sigma_j^\alpha\big] = 0$; thus the operator $\omega + \sum_l g_l\sigma_l^z\big/2$ $(l = A, B, C)$ is a conserved quantity and it can simply be treated as a constant with different values corresponding to the eigenvalues of $\sum_l g_l\sigma_l^z/2$. Then from equation (3), we have

$$H = \sum_\mu |\phi_\mu\rangle\langle\phi_\mu| \otimes H_E^{(\omega_\mu)}, \tag{5}$$

where the parameter ω_μ is given by $\omega_\mu = \omega + E_\mu$,and $H_E^{(\omega_\mu)}$ is given from H_E^ω by the replacement of ω with ω_μ, $H_E^{(\omega_\mu)} = H_E^\omega + E_\mu \sum_j \sigma_j^z/2$.

Considering the initial state $|\psi(0)\rangle = |\Phi_S(0)\rangle \otimes |\varphi_E(0)\rangle$, where $|\Phi_S(0)\rangle$ is the initial state for the three central spins and $|\varphi_E(0)\rangle$ is the initial state for the environmental Ising chain. The subsequent time evolution of the coupled spin system is determined by the time evolution operator $U(t) = \exp(-\mathrm{i}Ht)$. After explicitly knowing $|\psi(t)\rangle = U(t)|\psi(0)\rangle$, the main quantity of our investigation, i.e. the reduced density matrix of the tripartite system, will be straightforwardly obtained. For this purpose, we follow the standard procedure of Hamiltonian diagonalization by employing the Jordan–Wigner transformation [19] which maps spins to one-dimensional spinless fermions with the creation and annihilation operators $a_j^\dagger$ and a_j:

$$\begin{aligned} \sigma_j^x &= \prod_{i<j}(1 - 2a_i^\dagger a_i)(a_j + a_j^\dagger), \\ \sigma_j^y &= -\mathrm{i}\prod_{i<j}(1 - 2a_i^\dagger a_i)(a_j - a_j^\dagger), \\ \sigma_j^z &= 1 - 2a_j^\dagger a_j. \end{aligned} \tag{6}$$

After a straightforward transform, the dressed environmental Hamiltonian becomes

$$H_E^{(\omega_\mu)} = \sum_j^L [a_{j+1}a_j + a_{j+1}^\dagger a_j + a_j^\dagger a_{j+1} + a_j^\dagger a_{j+1}^\dagger + \omega_\mu(1 - 2a_j^\dagger a_j)]. \tag{7}$$

Now by introducing the Fourier transforms of the fermionic operators described by $d_k = \frac{1}{\sqrt{L}}\sum_j a_j\,\mathrm{e}^{-\mathrm{i}2\pi kj/L}$ with $k = -M, \ldots, M$ and $M = (L-1)/2$ for odd L, the Hamiltonian $H_E^{(\omega_\mu)}$ becomes

$$H_E^{(\omega_\mu)} = \sum_k \mathrm{i} d_k d_{-k} \sin\frac{2\pi k}{L} + \sum_k \mathrm{i} d_k^\dagger d_{-k}^\dagger \sin\frac{2\pi k}{L} + \sum_k d_k^\dagger d_k \left(2\cos\frac{2\pi k}{L} - 2\omega_\mu\right). \tag{8}$$

Finally by defining the Bogoliubov transformed fermion operators

$$\gamma_{k,\omega_\mu} = \cos\frac{\beta_k^{(\omega_\mu)}}{2} d_k - \mathrm{i}\sin\frac{\beta_k^{(\omega_\mu)}}{2} d_{-k}^\dagger, \tag{9}$$

with the angles $\beta_k^{(\omega_\mu)}$ satisfying

$$\beta_k^{(\omega_\mu)} = \arctan\left[\frac{\sin\frac{2\pi k}{L}}{\omega_\mu - \cos\frac{2\pi k}{L}}\right], \tag{10}$$

one can obtain the final Hamiltonian as

$$H_E^{(\omega_\mu)} = \sum_k \varepsilon_k^{(\omega_\mu)}\left(\gamma_{k,\omega_\mu}^\dagger \gamma_{k,\omega_\mu} - \frac{1}{2}\right), \tag{11}$$

where the energy spectrum $\varepsilon_k^{(\omega_\mu)}$ is expressed as follows:

$$\varepsilon_k^{(\omega_\mu)} = \sqrt{(2\omega_\mu - 2\cos ka)^2 + 4\sin^2 ka} \tag{12}$$

where the lattice spacing a takes the value of $2\pi/L$. According to equation (9), it is straightforward to find that the normal mode γ_{k,ω_μ} dressed by the system–environment interaction can be related to the purely environmental normal mode $\gamma_{k,\omega}$ by the relation

$$\begin{aligned} \gamma_{k,\omega_\mu} &= \left(\cos\Theta_k^{(\omega_\mu)}\right)\gamma_{k,\omega} - \mathrm{i}\left(\sin\Theta_k^{(\omega_\mu)}\right)\gamma_{-k,\omega}^\dagger, \\ \text{with}\quad \Theta_k^{(\omega_\mu)} &= \left(\beta_k^{(\omega_\mu)} - \beta_k^{(\omega)}\right)/2. \end{aligned} \tag{13}$$

The time evolution operator for the Hamiltonian (5) is given by

$$U(t) = \sum_{\mu=1}^{8} |\phi_\mu\rangle\langle\phi_\mu| \otimes U_E^{(\omega_\mu)}(t), \tag{14}$$

where $U_E^{(\omega_\mu)}(t) = \exp(-\mathrm{i}H_E^{(\omega_\mu)}t)$ is the projected time evolution operator for the Ising chain dressed by the system–environment interaction parameter ω_μ. In terms of these notations, we can derive the time evolution of quantum states and obtain the reduced density matrix of the system. Assume that initially the three-qubit system is disentangled with the environment, i.e. at $t = 0$ the three-qubit spins and the environmental Ising chain are assumed to be described by the product state $|\psi(0)\rangle = |\Phi_S(0)\rangle \otimes |\varphi_E(0)\rangle$, where $|\Phi_S(0)\rangle$ is the initial state for the three-qubit system and $|\varphi_E(0)\rangle = |0\rangle_{k=0} \otimes_{k\geqslant 0} |0\rangle_k|0\rangle_{-k}$ is the initial state for the environmental Ising chain, which is the vacuum of the fermionic modes described by $\gamma_k|0\rangle_k = 0$. Tracing out the environment, we can go straightforwardly to obtain the reduced density matrix of the three-qubit system:

$$\begin{aligned} \rho_S(t) &= \mathrm{Tr}_E|\psi(t)\rangle\langle\psi(t)| \\ &= \mathrm{Tr}_E[U(t)|\psi(0)\rangle\langle\psi(0)|U(t)^\dagger] \\ &= \mathrm{Tr}_E[\mathrm{e}^{-\mathrm{i}Ht}\rho_S(0) \otimes |\varphi_E(0)\rangle\langle\varphi_E(0)|\mathrm{e}^{iHt}] \\ &= \sum_{\mu,\nu=1}^{8} c_\mu c_\nu^* \langle\varphi_E(0)|U_E^{\dagger(\omega_\nu)}(t)U_E^{(\omega_\mu)}(t)|\varphi_E(0)\rangle|\phi_\mu\rangle\langle\phi_\nu|, \end{aligned} \tag{15}$$

with $c_\mu = \langle \phi_\mu | \Phi_S \rangle$. We define the decoherence factor as follows:

$$F(t) = \langle \varphi_E(0) | U_E^{\dagger(\omega_\nu)}(t) U_E^{(\omega_\mu)}(t) | \varphi_E(0) \rangle. \tag{16}$$

One can see from equation (16) that the decoherence factor $|F(t)_{\mu\nu}|$ can be considered as the amplitude of the overlap between two different states of the environment obtained by evolving the initial state with $H_E^{(\omega_\mu)}$ and $H_E^{(\omega_\nu)}$. According to equations (15) and (16), the environmental Ising chain only modulates the off-diagonal terms of $\rho_S(t)$, whereas the diagonal terms of $\rho_S(t)$ are not affected by the environment, because the decoherence factor remains unity when $\mu = \nu$; the fact means that there is no dynamic correlation between the central qubits and environmental Ising chain. Let the initial state of the three-qubit system be in a GHZ state $|\Phi_S(0)\rangle = \bar{a}_1|000\rangle + \bar{a}_8|111\rangle$, where $\bar{a}_1$ and $\bar{a}_8$ are the complex coefficients in polar forms $\bar{a}_1 = |\bar{a}_1|\mathrm{e}^{i\vartheta(\bar{a}_1)}$ and $\bar{a}_8 = |\bar{a}_8|\mathrm{e}^{i\vartheta(\bar{a}_8)}$, and they satisfy the normalization relation $|\bar{a}_1|^2 + |\bar{a}_8|^2 = 1$. Let us write the relative phase angle between the amplitudes $|\bar{a}_1|$ and $|\bar{a}_8|$ as $\delta = \vartheta(\bar{a}_1) - \vartheta(\bar{a}_8)$. From the evolution operator (14), the state vector of the composite system at time t is given by

$$|\psi(t)\rangle = \bar{a}_1|000\rangle \otimes U_E^{(\omega_1)}|\varphi_E(0)\rangle + \bar{a}_8|111\rangle \otimes U_E^{(\omega_8)}|\varphi_E(0)\rangle, \tag{17}$$

where $U_E^{(\omega_\mu)}$ $(\mu = 1, 8)$ can be obtained from the unitary operator $U(t)$ by replacing ω with ω_μ, respectively. When exposed to the environment, the initial GHZ class states evolve to the resultant state

$$\rho_{\mathrm{GHZ}}(t) = |\bar{a}_1|^2|000\rangle\langle 000| + \bar{a}_1\bar{a}_8^* F_{18}|000\rangle\langle 111| + \bar{a}_1^*\bar{a}_8 F_{18}^*|111\rangle\langle 000| + |\bar{a}_8|^2|111\rangle\langle 111|. \tag{18}$$

Equation (18) is our starting point for the following calculation and discussion. From the relationship between the Bogoliubov modes $\gamma_{k,\omega}$ and γ_{k,ω_μ} (equation (13)), the projected time evolution operator, and the initial vacuum state, the factor F_{18} in equation (16) which denotes the effect of decoherence induced by the environment can be written as [15]

$$\begin{aligned}
|F_{18}| &= \langle \varphi_E(0)|U_E^{\dagger(\omega_1)} U_E^{(\omega_8)}|\varphi_E(0)\rangle \\
&= {}_{-k}\langle 0|_k\langle 0|_{k>0} \otimes_{k=0} \langle 0| \prod_k \exp\left[\mathrm{i}t\varepsilon_k^{(\omega_1)}\left(\gamma_{k,\omega_1}^\dagger \gamma_{k,\omega_1} - \frac{1}{2}\right)\right] \\
&\quad \times \prod_k \exp\left[-\mathrm{i}\varepsilon_k^{(\omega_8)} t\left(\gamma_{k,\omega_8}^\dagger \gamma_{k,\omega_8} - \frac{1}{2}\right)\right]|0\rangle_{k=0} \otimes |0\rangle_k|0\rangle_{-k} \\
&= \prod_{k>0}\{1 - \sin^2\left(\varepsilon_k^{(\omega_1)}t\right)\sin^2\left(\varepsilon_k^{(\omega_8)}t\right) \\
&\quad \times \sin^2\left(\beta_k^{(\omega_1)} - \beta_k^{(\omega_8)}\right) - \left[\sin\left(\varepsilon_k^{(\omega_1)}t\right)\cos\left(\varepsilon_k^{(\omega_8)}t\right)\right. \\
&\quad \left.\times \sin\left(\beta_k^{(\omega_1)}\right) - \cos\left(\varepsilon_k^{(\omega_1)}t\right)\sin\left(\varepsilon_k^{(\omega_8)}t\right)\sin\left(\beta_k^{(\omega_8)}\right)\right]^2\}^{1/2} \\
&= \prod_{k>0} F_k,
\end{aligned} \tag{19}$$

where $\beta_k^{(\omega_\mu)}$, $\varepsilon_k^{(\omega_\mu)}$ $(\mu = 1, 8)$ can be calculated by replacing ω with ω_μ in equations (10) and (12), respectively, and ω_μ is expressed as follows:

$$\begin{aligned}
\omega_1 &= \omega + E_1 = \omega + \frac{g_A + g_B + g_C}{2}, \\
\omega_8 &= \omega + E_8 = \omega - \frac{g_A + g_B + g_C}{2}.
\end{aligned} \tag{20}$$

Clearly, every factor F_k is less than unity, so it can be expected that in the large L limit, $|F_{18}|$ will go to zero under some reasonable conditions.

3. Bell-nonlocality sudden death

It is known that except for the two-qubit case, there is still no efficient general method to evaluate all entanglement for general mixed states of multipartite systems. The tool employed here to analyze the tripartite nonlocality is the Bell-type inequality, which is one of the important mathematical criteria for entanglement.

For $N-$qubit systems, there are two kinds of multipartite Bell inequalities used to detect the degree of nonlocality as measured by the extent of their violations. The first one was proposed by Mermin [20] and further developed by Ardehali [21], Belinskii and Klyshko [22] (MABK inequality) and Gisin and Bechmann-Pasquinucci [23]. The second one was presented later by Werner and Wolf [24] and Żukowski and Brukner [25] (WWZB inequality). This single general inequality is both a necessary and a sufficient condition for the behavior of a quantum system of N qubits to be describable by a fully Bell-local hidden-variable model; however, it is often too difficult to calculate. As these two inequalities are equivalent in the case $N = 3$, we choose the MABK inequality as a criterion for Bell locality, because its computation is much easier than the second one.

A quantum state ρ violates the MABK inequality if

$$|\langle B_N\rangle_\rho| > 1. \tag{21}$$

In the simplest case of two qubits, $N = 2$, the operator B_N on the left-hand side of the MABK inequality is just the Clauser–Horne—Shimony–Holt (CHSH) operator for two-particle systems [26]:

$$B_2 = \tfrac{1}{2}[M_A M_B + M_A M'_B + M'_A M_B - M'_A M'_B]. \tag{22}$$

The specific form of the operator B_3 for our tripartite system can be written as

$$B_3 = \tfrac{1}{2}[M_A M_B M'_C + M_A M'_B M_C + M'_A M_B M_C - M'_A M'_B M'_C], \tag{23}$$

where the measurement operators M_K and M'_K are the operators corresponding to measurements on each of the qubits K (A B, or C), while the primed and unprimed terms denote the two different directions in which the measurements on each particle can be chosen. Defining $M_A \equiv \sigma_y$ and $M'_A \equiv \sigma_x$, the measurement operator acting upon each successive subsystem is defined with respect to the first qubit by θ_K:

$$\begin{pmatrix} M_K \\ M'_K \end{pmatrix} = R(\theta_K)\begin{pmatrix} M_A \\ M'_A \end{pmatrix}, \tag{24}$$

where

$$R(\theta_K) = \begin{pmatrix} \cos\theta_K & -\sin\theta_K \\ \sin\theta_K & \cos\theta_K \end{pmatrix}. \tag{25}$$

In the case of three qubits, there are two such rotation angles θ_B and θ_C; the corresponding measurement operators for qubits A, B and C can be written in terms of Pauli operators as

$$\begin{aligned}
M_A &= \sigma_y \otimes I \otimes I, \\
M'_A &= \sigma_x \otimes I \otimes I, \\
M_B &= I \otimes [\cos(\theta_B)\sigma_y - \sin(\theta_B)\sigma_x] \otimes I, \\
M'_B &= I \otimes [\sin(\theta_B)\sigma_y + \cos(\theta_B)\sigma_x] \otimes I, \\
M_C &= I \otimes I \otimes [\cos(\theta_C)\sigma_y - \sin(\theta_C)\sigma_x], \\
M'_C &= I \otimes I \otimes [\sin(\theta_C)\sigma_y + \cos(\theta_C)\sigma_x].
\end{aligned} \tag{26}$$

The expectation value of the B_3 operator for the GHZ state can be obtained as

$$\begin{aligned}\langle B_3\rangle_{\rho_{\mathrm{GHZ}}(t)} &= \mathrm{tr}[B_3\rho_{\mathrm{GHZ}}(t)]\\ &= \mathrm{tr}\left[\tfrac{1}{2}(M_A M_B M_C' + M_A M_B' M_C + M_A' M_B M_C - M_A' M_B' M_C')\rho_{\mathrm{GHZ}}(t)\right]\\ &= -2F_{18}[(\bar{a}_1\bar{a}_8^* + \bar{a}_1^*\bar{a}_8)\cos(\theta_B+\theta_C) + \mathrm{i}(\bar{a}_1\bar{a}_8^* - \bar{a}_1^*\bar{a}_8)\sin(\theta_B+\theta_C)]\\ &= -4|\bar{a}_1|\cdot|\bar{a}_8|\cdot F_{18}\cos\theta_{BC\delta},\end{aligned} \tag{27}$$

where $\theta_{BC\delta} = \theta_B + \theta_C + \delta$. The MABK inequality is violated whenever $|\langle B_3\rangle_{\rho_{\mathrm{GHZ}}(t)}| > 1$. One can calculate the expectation value of the MABK operator by substituting equation (19) into the above equation. It can be seen that the maximum value is $|\langle B_3\rangle_{\rho_{\mathrm{GHZ}}(t)}| = 2F_{18}$, since the cosine term is limited by unity, and for the states of interest $1/2 \geqslant |\bar{a}_1||\bar{a}_8| > 1/4$, where the upper bound $1/2$ represents a maximally entangled state and the lower bound $1/4$ corresponds to a maximally mixed state. When the time is short enough, inequality (21) is satisfied, that is, Bell nonlocality is existent at the beginning. According to equation (19), each factor F_k has a norm less than unity; therefore, $|F_{18}|$ may decrease to zero in the large L limit under some reasonable conditions. It means that under some reasonable conditions, the expectation value of the MABK operator becomes equal to or less than unity in a finite timescale τ_{BNSD}, so that tripartite Bell nonlocality is nonexistent after that. The timescale τ_{BNSD} suffices to demonstrate tripartite BNSD of initial Bell-nonlocal states due to the correlated environment. When $t > \tau_{\mathrm{BNSD}}$, $|\langle B_3\rangle_{\rho_{\mathrm{GHZ}}(t)}|$ is less than unity and no longer violates the inequality for all choices of $\theta_{BC\delta}$.

To analyze the effect of an environment on Bell nonlocality in the three-qubit system, we numerically calculate the exact expression of equation (27) and plot the expectation value of the MABK operator as a function of time for different values of the transverse field and the coupling constant $g = (g_A + g_A + g_C)/2$ in figure 1. Under weak coupling ($g = 0.15$), one can find that the expectation value of the operator shows an oscillating and exponentially decaying function when $\omega < 2$ (for the curves $\omega = 0$ and $\omega = 1$). The phenomenon of BNSD occurs for definite times and then the nonlocality revives. This process will persist for some periods determined by the parameter ω. However, when $\omega \geqslant 2$, the evolution of $|\langle B_3\rangle_{\rho_{\mathrm{GHZ}}(t)}|$ will vanish monotonically without any revivals, and it is very sensitive to the changes of the transverse field. With the decrease of the strength of the transverse field, the expectation value of the operator decays more sharply. For the case $g = 1.5$, one can find that the expectation value of the operator will no longer revived, but will decay exponentially to zero. Therefore, BNSD can always occur by the timescale for which $|\langle B_3\rangle_{\rho_{\mathrm{GHZ}}(t)}|$ reaches unity from above. When we continue enlarging the coupling strength, e.g. $g = 15$, some collapses and revivals are observed in the time evolution of the expectation value of the operator. Moreover, we can find that there exists the destruction of Bell-nonlocality behavior at the very beginning, i.e. $|\langle B_3\rangle_{\rho_{\mathrm{GHZ}}(0)}| > 1$. Subsequently, the phenomenon of BNSD occurs at some definite timescale and the Bell nonlocality revives later; only when the time reaches a certain value can the tripartite nonlocality not revive eventually. If the coupling takes a value large enough as $g = 150$, it can be seen from the last subfigure of figure 1 that the expectation value of the operator oscillates quickly and tends to be a constant near 1.95 with the increase of time. Therefore, inequality (19) is always satisfied, $|\langle B_3\rangle_{\rho_{\mathrm{GHZ}}(t)}| > 1$, and it can be expected that the phenomenon of BNSD will not occur in the strong-coupling region.

Such a result is consistent with the analysis of [16] in which the authors discuss the entanglement dynamics of three-qubit states under decoherence induced by an environment. They employ negativity to measure the quantum correlation between one central qubit and other two qubits $N_{\mathrm{A-BC}}$, $N_{\mathrm{B-AC}}$ and $N_{\mathrm{C-AB}}$. In our presentation, we aim to study Bell nonlocality in a three-qubit system coupled to an environment by making use of the

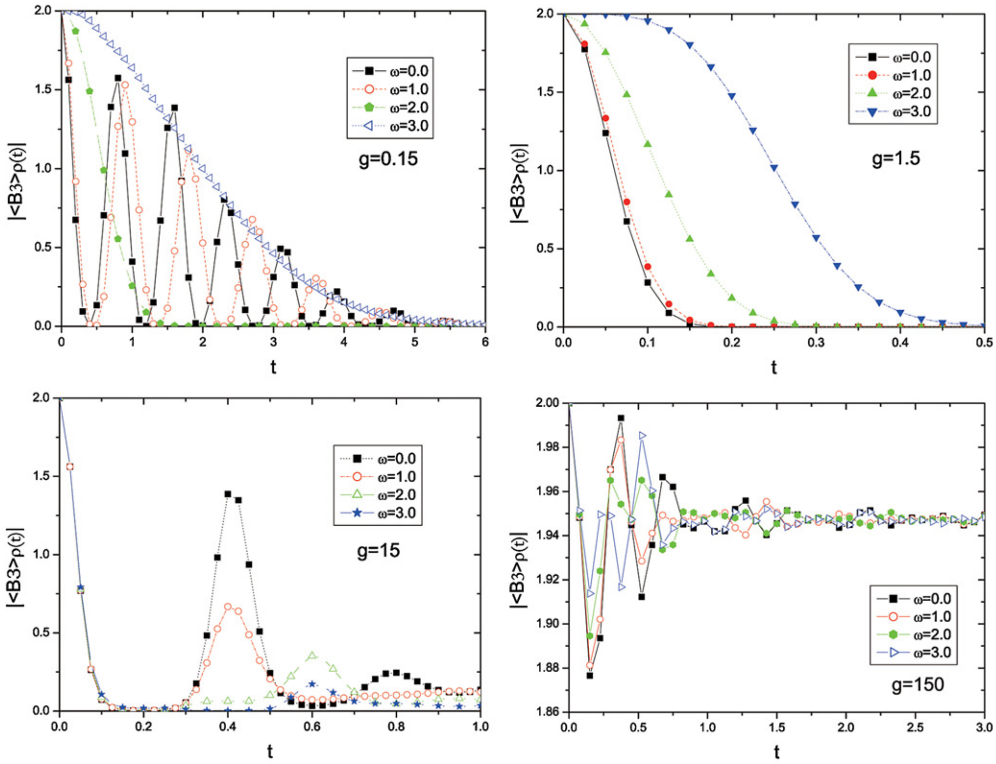

Figure 1. The absolute value of $\langle B_3\rangle_{\rho_{\mathrm{GHZ}}(t)}$ versus time is plotted for four different kinds of coupling: $g = 0.15$, $g = 1.5$, $g = 15$ and $g = 150$ when ω as the strength of the transverse field of the environment takes different values, where $\theta_{BC\delta} = \pi$, $|\bar{a}_1| = |\bar{a}_8| = 1/\sqrt{2}$, $g = (g_A+g_B+g_C)/2$ and $L = 200$.

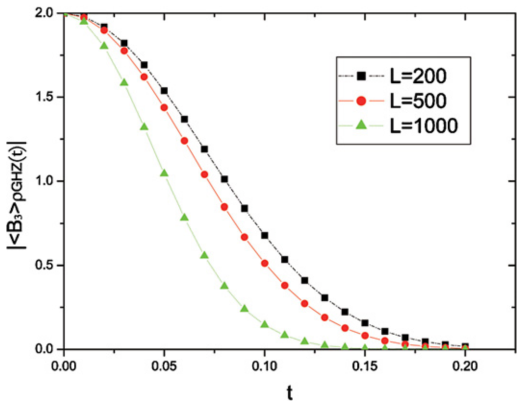

Figure 2. The absolute value of $\langle B_3\rangle_{\rho_{\mathrm{GHZ}}(t)}$ versus time is plotted under different sizes of the degrees of freedom of the environment, where $\theta_{BC\delta} = \pi$, $|\bar{a}_1| = |\bar{a}_8| = 1/\sqrt{2}$, $g = (g_A + g_B + g_C)/2 = 1.5$ and $\omega = 2$.

MABK operator, and obtain some similar results compared with their figure 2. It is not surprising because negativity can efficiently calculate the quantum entanglement of the three-qubit quantum state, while the MABK inequality is a valid probe of entanglement: given a source of entangled particles, its violation is a signature of the nonlocal nature of quantum mechanics.

In order to examine the effect of the size of the degrees of freedom of an environment on Bell nonlocality, we numerically calculate the evolution of $|\langle B_3\rangle_{\rho_{\mathrm{GHZ}}(t)}|$ with different sizes of the environmental chain with the coupling strength $g = 1.5$ in figure 2. From the figure one can see that the expectation of the operator decays exponentially to zero with the increase of time. Thus, BNSD can always take place at some definite timescale τ_{BNSD}; namely, there exists a finite time after which we have $|\langle B_3\rangle_{\rho_{\mathrm{GHZ}}(\tau_{\mathrm{BNSD}})}| \leqslant 1$. With the increase of the size L from 200 to 1000, we can observe the decrease of the timescale. In respect to the fact that each factor F_k in equation (19) is less than unity, it is reasonable to conclude that the larger size of the environment will more effectively suppress the factor F_{18}, and consequently suppress the expectation value of the MABK operator.

4. Conclusions

We have studied the nonlocal behavior of a three-qubit system coupled to a transverse Ising chain by calculating the extent of violation of MABK inequality. Our results imply that there exists the destruction of Bell-nonlocal behavior due to the environment.

In the weak-coupling region, the phenomenon of BNSD is shown by the qualitative demonstration that the three-qubit system prepared in the GHZ state initially violating the MABK inequality fails to violate it in a finite timescale. The effect of the strength of the transverse field and the size of the environmental Ising chain on the expectation value of the Bell-type operator B_3 is also considered. Under different conditions, i.e. with different coupling strength, transverse field and the size of the environmental chain, the expectation value of the Bell-type operator for the state under the influence of a spin environment on the three-qubit system prepared in the GHZ state shows different behaviors. However in the strong-coupling region, we find that the inequality $|\langle B_3\rangle_{\rho_{\mathrm{GHZ}}(t)}| > 1$ is always satisfied, which means that tripartite Bell-nonlocality is existent all the time and one cannot observe the effect of BNSD.

The study of BNSD is of importance in a tripartite system, because tripartite systems can exhibit some essential characteristics which are impossible in the bipartite case. In addition, our results may help to understand the Bell-nonlocal behavior of a tripartite system in a correlated environment.

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